

# **Nerovnice v součinném a podílovém tvaru**

**V množině Z řešte nerovnice:**

$$1) \frac{2(3x-1)}{x+4} \leq 0$$

$$2) \frac{x^2 + 4x + 5}{x(x-1)} > 0$$

$$3) \frac{x^2 - 6x + 8}{2x - 9} < 0$$

$$4) \frac{(x^2 + 4x + 4)(x^2 - 4)}{x - 4} \geq 0$$

**V množině R řešte nerovnice:**

$$5) \frac{x+1}{3x-2} < 2$$

$$6) \frac{2x-15}{3x+7} < \frac{1}{2}$$

$$7) \frac{7x-5}{8x+3} > 4$$

$$8) \frac{3-4x}{3x-4} \geq 2$$

$$9) \frac{2x-1}{x+1} \geq 1$$

$$10) \frac{x-7}{x-1} \leq 9$$

$$11) \frac{x+3}{3x+3} < 3$$

$$12) \frac{x+2}{x-4} < 2$$

$$13) (5-4x)(3^x - 9) > 0$$

$$14) \frac{x^2 - x + 1}{x^2 + x + 1} \geq \frac{1}{3}$$

$$15) \frac{5x^2 - 2}{4x^2 - x + 3} < 1$$

$$16) \frac{1}{2x+5} - \frac{2}{7x+4} \geq 0$$

$$17) \frac{1}{x+1} < \frac{1}{3x-2}$$

$$18) \frac{3}{x+2} + \frac{2}{x-3} \geq 0$$

$$19) \frac{x+1}{x+3} > \frac{x+5}{x+6}$$

$$20) \frac{2x-1}{x-1} - \frac{x+2}{x+1} \geq 0$$

$$21) \frac{x-1}{x+1} - \frac{1+x}{1-x} < 0$$

$$22) \frac{5}{x+2} < \frac{10}{x-1}$$

$$23) \frac{x}{x-1} - \frac{2}{x+1} - \frac{8}{x^2-1} < 0$$

$$24) \frac{x+3}{x+5} - \frac{x+2}{x+1} > 0$$

$$25) \frac{x}{x-2} - \frac{3}{x+1} \leq 1$$

$$26) \frac{(5^x - 25)(2x - 5)}{4x + 3} > 0$$

$$27) \frac{(\log x - 1)(3x - 6)}{(2^x - 4)(1 - x)} \geq 0$$

$$28) (x + 2)(2x - 1)(5 - 6x) \geq 0$$

$$29) \frac{(8 - x)(2x - 5)}{2x^2 + 3x - 2} \geq 0$$

$$30) (2^x + 1)(1 - 3x)(x^2 - 1) \leq 0$$

$$31) (3x^2 + 4x + 5) \left( 2^x - \frac{1}{2} \right) (9x^2 - 1) \geq 0$$

$$32) \frac{-1 + \log x}{(3 - 3^x)(x - 4)} \geq 0$$

$$33) \frac{\ln x}{(2^x - 8)(x - 5)} \leq 0$$

$$34) \frac{x-1}{x+2} > \frac{x+3}{x-2}$$

$$35) \frac{2x+3}{x-6} < \frac{x+9}{x-7}$$

$$36) \frac{3}{x+1} + \frac{7}{x+2} < \frac{6}{x-1}$$

$$37) \frac{5-x}{3-x} < \frac{3x-1}{2-x}$$

$$38) \frac{7}{(x-2)(x-3)} + \frac{9}{x-3} + 1 < 0$$

$$39) 2 - \frac{x-3}{x-2} \geq \frac{x-2}{x-1}$$

$$40) \frac{2(x-4)}{(x-1)(x-7)} \geq \frac{1}{x-2}$$

$$41) \frac{x-1}{x+1} + \frac{x+1}{x-1} > \frac{10}{3}$$

$$42) \frac{2x}{x^2-9} \leq \frac{1}{x+2}$$

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$$43) \frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$$

$$44) \frac{(2-x^2)(x-3)^3}{(x+1)(x^2-3x-4)} \geq 0$$

$$45) \frac{x^4 - 3x^3 + 2x^2}{x^2 - x - 30} > 0$$

$$46) \frac{4x^2 - 5x - 1}{2x^2 - 5x + 3} < 1$$

$$47) \frac{1}{x-8} + \frac{1}{x-6} + \frac{1}{x+6} + \frac{1}{x+8} > 0$$

$$48) -\frac{2}{3} < \frac{3-4x}{5x+2} < \frac{3}{2}$$

$$49) 1 < \frac{3x^2 - 7x + 8}{x^2 + 1} \leq 2$$

$$50) 1 < \frac{2x^2 - 7x - 29}{x^2 - 2x - 15} < 2$$

**V množině Z řešte nerovnice:**

$$1) \frac{2(3x-1)}{x+4} \leq 0$$

$$[(3x-1) \geq 0 \wedge (x+4) < 0] \vee [(3x-1) \leq 0 \wedge (x+4) > 0]$$

$$\left(x \geq \frac{1}{3} \wedge x < -4\right) \vee \left(x \leq \frac{1}{3} \wedge x > -4\right)$$

$$x \in \{-3; -2; -1; 0\}$$

$$2) \frac{x^2 + 4x + 5}{x(x-1)} > 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{-4}}{2} = \emptyset \Rightarrow x^2 + 4x + 5 > 0$$

$$x(x-1) > 0$$

$$[x > 0 \wedge (x-1) > 0] \vee [x < 0 \wedge (x-1) < 0]$$

$$(x > 0 \wedge x > 1) \vee (x < 0 \wedge x < 1)$$

$$x \in Z - \{0; 1\}$$

$$3) \frac{x^2 - 6x + 8}{2x - 9} < 0$$

$$x_{1,2} = \frac{6 \pm 2}{2} \rightarrow x_1 = 4, x_2 = 2$$

$$\frac{(x-2)(x-4)}{2x-9} < 0$$

	$(-\infty; 2)$	$(2; 4)$	$(4; 4,5)$	$(4,5; \infty)$
$(x-2)$	-	+	+	+
$(x-4)$	-	-	+	+
$(2x-9)$	-	-	-	+
	-	+	-	+

$$x \in (-\infty; 2) \cup \left(4; \frac{9}{2}\right)$$

$$x \in \{\dots; -1; 0; 1\}$$

$$4) \frac{(x^2 + 4x + 4)(x^2 - 4)}{x - 4} \geq 0$$

$$\frac{(x+2)^2(x+2)(x-2)}{x-4} \geq 0$$

	$(-\infty; -2)$	$\langle -2; 2 \rangle$	$\langle 2; 4 \rangle$	$(4; \infty)$
$(x+2)$	-	+	+	+
$(x-2)$	-	-	+	+
$(x-4)$	-	-	-	+
	-	+	-	+

$$x \in \langle -2; 2 \rangle \cup (4; \infty)$$

$$x \in \{-2; -1; 0; 1; 2; 5; 6; \dots\}$$

**V množině R řešte nerovnice:**

$$5) \quad \frac{x+1}{3x-2} < 2$$

$$\frac{x+1-6x+4}{3x-2} < 0$$

$$\frac{-5x+5}{3x-2} < 0$$

$$[(-5x+5) > 0 \wedge (3x-2) < 0] \vee [(-5x+5) < 0 \wedge (3x-2) > 0]$$

$$\left(x < 1 \wedge x < \frac{2}{3}\right) \vee \left(x > 1 \wedge x > \frac{2}{3}\right)$$

$$x \in \left(-\infty; \frac{2}{3}\right) \cup (1; \infty)$$

$$6) \quad \frac{2x-15}{3x+7} < \frac{1}{2}$$

$$\frac{4x-30-3x-7}{6x+14} < 0$$

$$\frac{x-37}{6x+14} < 0$$

$$[(x-37) > 0 \wedge (6x+14) < 0] \vee [(x-37) < 0 \wedge (6x+14) > 0]$$

$$\left(x > 37 \wedge x < -\frac{7}{3}\right) \vee \left(x < 37 \wedge x > -\frac{7}{3}\right)$$

$$x \in \left(-\frac{7}{3}; 37\right)$$

$$7) \quad \frac{7x-5}{8x+3} > 4$$

$$\frac{7x-5-32x-12}{8x+3} > 0$$

$$\frac{-25x-17}{8x+3} > 0$$

$$[(-25x-17) > 0 \wedge (8x+3) > 0] \vee [(-25x-17) < 0 \wedge (8x+3) < 0]$$

$$\left(x < -\frac{17}{25} \wedge x > -\frac{3}{8}\right) \vee \left(x > -\frac{17}{25} \wedge x < -\frac{3}{8}\right)$$

$$x \in \left(-\frac{17}{25}; -\frac{3}{8}\right)$$

$$\begin{aligned}
 8) \quad & \frac{3-4x}{3x-4} \geq 2 \\
 & \frac{3-4x-6x+8}{3x-4} \geq 0 \\
 & \frac{11-10x}{3x-4} \geq 0 \\
 & [(11-10x) \geq 0 \wedge (3x-4) > 0] \vee [(11-10x) \leq 0 \wedge (3x-4) < 0] \\
 & \left(x \leq \frac{11}{10} \wedge x > \frac{4}{3}\right) \vee \left(x \geq \frac{11}{10} \wedge x < \frac{4}{3}\right) \\
 & x \in \left\langle \frac{11}{10}; \frac{4}{3} \right\rangle
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & \frac{2x-1}{x+1} \geq 1 \\
 & \frac{2x-1-x-1}{x+1} \geq 0 \\
 & \frac{x-2}{x+1} \geq 0 \\
 & [(x-2) \geq 0 \wedge (x+1) > 0] \vee [(x-2) \leq 0 \wedge (x+1) < 0] \\
 & (x \geq 2 \wedge x > -1) \vee (x \leq 2 \wedge x < -1) \\
 & x \in (-\infty; -1) \cup \langle 2; \infty)
 \end{aligned}$$

$$\begin{aligned}
 10) \quad & \frac{x-7}{x-1} \leq 9 \\
 & \frac{x-7-9x+9}{x-1} \leq 0 \\
 & \frac{-8x+2}{x-1} \leq 0 \\
 & \frac{2(-4x+1)}{x-1} \leq 0 \\
 & [(-4x+1) \geq 0 \wedge (x-1) < 0] \vee [(-4x+1) \leq 0 \wedge (x-1) > 0] \\
 & \left(x \leq \frac{1}{4} \wedge x < 1\right) \vee \left(x \geq \frac{1}{4} \wedge x > 1\right) \\
 & x \in \left(-\infty; \frac{1}{4}\right) \cup (1; \infty)
 \end{aligned}$$

$$\begin{aligned}
 11) \quad & \frac{x+3}{3x+3} < 3 \\
 & \frac{x+3-9x-9}{3(x+1)} < 0 \\
 & -\frac{2(4x+3)}{3(x+1)} < 0
 \end{aligned}$$

$$\frac{x+3-9x-9}{3(x+1)} < 0$$

$$-\frac{2(4x+3)}{3(x+1)} < 0$$

$$[(4x+3) > 0 \wedge (x+1) > 0] \vee [(4x+3) < 0 \wedge (x+1) < 0]$$

$$\left(x > -\frac{3}{4} \wedge x > -1\right) \vee \left(x < -\frac{3}{4} \wedge x < -1\right)$$

$$x \in (-\infty; -1) \cup \left(-\frac{3}{4}; \infty\right)$$

12)  $\frac{x+2}{x-4} < 2$

$$\frac{x+2-2x+8}{x-4} < 0$$

$$\frac{-x+10}{x-4} < 0$$

$$[(-x+10) < 0 \wedge (x-4) > 0] \vee [(-x+10) > 0 \wedge (x-4) < 0]$$

$$(x > 10 \wedge x > 4) \vee (x < 10 \wedge x < 4)$$

$$x \in (-\infty; 4) \cup (10; \infty)$$

13)  $(5-4x)(3^x-9) > 0$

$$[(5-4x) > 0 \wedge (3^x-9) > 0] \vee [(5-4x) < 0 \wedge (3^x-9) < 0]$$

$$\left(x < \frac{5}{4} \wedge x > 2\right) \vee \left(x > \frac{5}{4} \wedge x < 2\right)$$

$$x \in \left(\frac{5}{4}; 2\right)$$

14)  $\frac{x^2-x+1}{x^2+x+1} \geq \frac{1}{3}$

$$\frac{3x^2-3x+3-x^2-x-1}{3(x^2+x+1)} \geq 0$$

$$\frac{2x^2-4x+2}{3(x^2+x+1)} \geq 0$$

$$\frac{2(x-1)^2}{3(x^2+x+1)} \geq 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{-3}}{2} = \emptyset \Rightarrow x^2+x+1 > 0$$

$$(x-1)^2 \geq 0$$

$$x \in (-\infty; \infty)$$

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$$15) \frac{5x^2 - 2}{4x^2 - x + 3} < 1$$

$$\frac{5x^2 - 2 - 4x^2 + x - 3}{4x^2 - x + 3} < 0$$

$$\frac{x^2 + x - 5}{4x^2 - x + 3} < 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{-47}}{8} = \emptyset \Rightarrow 4x^2 - x + 3 > 0$$

$$x_{3,4} = \frac{-1 \pm \sqrt{21}}{2}$$

$$\frac{\left(x - \frac{-1 + \sqrt{21}}{2}\right)\left(x - \frac{-1 - \sqrt{21}}{2}\right)}{4x^2 - x + 3} < 0$$

	$\left(-\infty; \frac{-1 - \sqrt{21}}{2}\right)$	$\left(\frac{-1 - \sqrt{21}}{2}; \frac{-1 + \sqrt{21}}{2}\right)$	$\left(\frac{-1 + \sqrt{21}}{2}; \infty\right)$
$\left(x + \frac{1 - \sqrt{21}}{2}\right)$	-	-	+
$\left(x + \frac{1 + \sqrt{21}}{2}\right)$	-	+	+
	+	-	+

$$x \in \left(-\frac{1 + \sqrt{21}}{2}; \frac{-1 + \sqrt{21}}{2}\right)$$

$$16) \frac{1}{2x+5} - \frac{2}{7x+4} \geq 0$$

$$\frac{7x+4-4x-10}{(2x+5)(7x+4)} \geq 0$$

$$\frac{3(x-2)}{(2x+5)(7x+4)} \geq 0$$

	$\left(-\infty; -\frac{5}{2}\right)$	$\left(-\frac{5}{2}; -\frac{4}{7}\right)$	$\left(-\frac{4}{7}; 2\right)$	$\langle 2; \infty$
$(x-2)$	-	-	-	+
$(2x+5)$	-	+	+	+
$(7x+4)$	-	-	+	+
	-	+	-	+

$$x \in \left(-\frac{5}{2}; -\frac{4}{7}\right) \cup \langle 2; \infty$$

$$17) \frac{1}{x+1} < \frac{1}{3x-2}$$

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$$\frac{3x-2-x-1}{(x+1)(3x-2)} < 0$$

$$\frac{2x-3}{(x+1)(3x-2)} < 0$$

	$(-\infty; -1)$	$\left(-1; \frac{2}{3}\right)$	$\left(\frac{2}{3}; \frac{3}{2}\right)$	$\left(\frac{3}{2}; \infty\right)$
$(2x-3)$	-	-	-	+
$(x+1)$	-	+	+	+
$(3x-2)$	-	-	+	+
	-	+	-	+

$$x \in (-\infty; -1) \cup \left(\frac{2}{3}; \frac{3}{2}\right)$$

18)  $\frac{3}{x+2} + \frac{2}{x-3} \geq 0$

$$\frac{3x-9+2x+4}{(x+2)(x-3)} \geq 0$$

$$\frac{5x-5}{(x+2)(x-3)} \geq 0$$

$$\frac{5(x-1)}{(x+2)(x-3)} \geq 0$$

	$(-\infty; -2)$	$(-2; 1)$	$(1; 3)$	$(3; \infty)$
$(x-1)$	-	-	+	+
$(x+2)$	-	+	+	+
$(x-3)$	-	-	-	+
	-	+	-	+

$$x \in (-2; 1) \cup (3; \infty)$$

19)  $\frac{x+1}{x+3} > \frac{x+5}{x+6}$

$$\frac{(x+1)(x+6) - (x+5)(x+3)}{(x+3)(x+6)} > 0$$

$$\frac{x^2 + 7x + 6 - x^2 - 8x - 15}{(x+3)(x+6)} > 0$$

$$-\frac{x+9}{(x+3)(x+6)} > 0$$

	$(-\infty; -9)$	$(-9; -6)$	$(-6; -3)$	$(-3; \infty)$
$(x+9)$	-	+	+	+
$(x+3)$	-	-	-	+
$(x+6)$	-	-	+	+
	+	-	+	-

$$x \in (-\infty; -9) \cup (-6; -3)$$

$$20) \frac{2x-1}{x-1} - \frac{x+2}{x+1} \geq 0$$

$$\frac{(2x-1)(x+1) - (x+2)(x-1)}{(x-1)(x+1)} \geq 0$$

$$\frac{2x^2 + x - 1 - x^2 - x + 2}{(x-1)(x+1)} \geq 0$$

$$\frac{x^2 + 1}{(x-1)(x+1)} \geq 0$$

$$x^2 + 1 > 0$$

	$(-\infty; -1)$	$(-1; 1)$	$(1; \infty)$
$(x-1)$	-	-	+
$(x+1)$	-	+	+
	+	-	+

$$x \in (-\infty; -1) \cup (1; \infty)$$

$$21) \frac{x-1}{x+1} - \frac{1+x}{1-x} < 0$$

$$\frac{(x-1)^2 + (x+1)^2}{(x+1)(x-1)} < 0$$

$$\frac{x^2 - 2x + 1 + x^2 + 2x + 1}{(x+1)(x-1)} < 0$$

$$\frac{2(x^2 + 1)}{(x-1)(x+1)}$$

$$x^2 + 1 > 0$$

	$(-\infty; -1)$	$(-1; 1)$	$(1; \infty)$
$(x-1)$	-	-	+
$(x+1)$	-	+	+
	+	-	+

$$x \in (-1; 1)$$

$$22) \frac{5}{x+2} < \frac{10}{x-1}$$

$$\frac{5x-5-10x-20}{(x+2)(x-1)} < 0$$

$$-\frac{5(x+5)}{(x+2)(x-1)} < 0$$

	$(-\infty; -5)$	$(-5; -2)$	$(-2; 1)$	$(1; \infty)$
$(x+5)$	-	+	+	+
$(x+2)$	-	-	+	+
$(x-1)$	-	-	-	+
	+	-	+	-

$$x \in (-5; -2) \cup (1; \infty)$$

$$23) \frac{x}{x-1} - \frac{2}{x+1} - \frac{8}{x^2-1} < 0$$

$$\frac{x^2 + x - 2x + 2 - 8}{(x-1)(x+1)} < 0$$

$$\frac{x^2 - x - 6}{(x+1)(x-1)} < 0$$

$$x_{1,2} = \frac{1 \pm 5}{2} \rightarrow x_1 = 3, x_2 = -2$$

$$\frac{(x+2)(x-3)}{(x-1)(x+1)} < 0$$

	$(-\infty; -2)$	$(-2; -1)$	$(-1; 1)$	$(1; 3)$	$(3; \infty)$
$(x+2)$	-	+	+	+	+
$(x-3)$	-	-	-	-	+
$(x-1)$	-	-	-	+	+
$(x+1)$	-	-	+	+	+
	+	-	+	-	+

$$x \in (-2; -1) \cup (1; 3)$$

$$24) \frac{x+3}{x+5} - \frac{x+2}{x+1} > 0$$

$$\frac{(x+3)(x+1) - (x+2)(x+5)}{(x+5)(x+1)} > 0$$

$$\frac{x^2 + 4x + 3 - x^2 - 7x - 10}{(x+5)(x+1)} > 0$$

$$-\frac{3x+7}{(x+5)(x+1)} > 0$$

	$(-\infty; -5)$	$\left(-5; -\frac{7}{3}\right)$	$\left(-\frac{7}{3}; -1\right)$	$(-1; \infty)$
$(3x+7)$	-	-	+	+
$(x+5)$	-	+	+	+
$(x+1)$	-	-	-	+
	+	-	+	-

$$x \in (-\infty; -5) \cup \left(-\frac{7}{3}; -1\right)$$

$$25) \frac{x}{x-2} - \frac{3}{x+1} \leq 1$$

$$\frac{x(x+1) - 3(x-2) - (x-2)(x+1)}{(x-2)(x+1)} \leq 0$$

$$\frac{x^2 + x - 3x + 6 - x^2 + x + 2}{(x-2)(x+1)} \leq 0$$

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$$\frac{-x+8}{(x-2)(x+1)} \leq 0$$

	$(-\infty; -1)$	$(-1; 2)$	$(2; 8)$	$\langle 8; \infty$
$(-x+8)$	+	+	+	-
$(x-2)$	-	-	+	+
$(x+1)$	-	+	+	+
	+	-	+	-

$$x \in (-1; 2) \cup \langle 8; \infty$$

$$26) \frac{(5^x - 25)(2x - 5)}{4x + 3} > 0$$

	$(-\infty; -\frac{3}{4})$	$(-\frac{3}{4}; 2)$	$(2; \frac{5}{2})$	$(\frac{5}{2}; \infty)$
$(5^x - 25)$	-	-	+	+
$(2x-5)$	-	-	-	+
$(4x+3)$	-	+	+	+
	-	+	-	+

$$x \in \left(-\frac{3}{4}; 2\right) \cup \left(\frac{5}{2}; \infty\right)$$

$$27) \frac{(\log x - 1)(3x - 6)}{(2^x - 4)(1 - x)} \geq 0$$

	$(-\infty; 1)$	$(1; 2)$	$(2; 10)$	$\langle 10; \infty$
$(\log x - 1)$	-	-	-	+
$(3x - 6) = (2^x - 4)$	-	-	+	+
$(1-x)$	+	-	-	-
	-	+	+	-

$$x \in (1; 2) \cup (2; 10)$$

$$28) (x+2)(2x-1)(5-6x) \geq 0$$

	$(-\infty; -2)$	$\langle -2; \frac{1}{2} \rangle$	$\langle \frac{1}{2}; \frac{5}{6} \rangle$	$\langle \frac{5}{6}; \infty$
$(x+2)$	-	+	+	+
$(2x-1)$	-	-	+	+
$(5-6x)$	+	+	+	-
	+	-	+	-

$$x \in (-\infty; -2) \cup \left\langle \frac{1}{2}; \frac{5}{6} \right\rangle$$

$$29) \frac{(8-x)(2x-5)}{2x^2+3x-2} \geq 0$$

$$x_{1,2} = \frac{-3 \pm 5}{4} \rightarrow x_1 = -2, x_2 = \frac{1}{2}$$

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$$\frac{(8-x)(2x-5)}{(x+2)\left(x-\frac{1}{2}\right)} \geq 0$$

	$(-\infty; -2)$	$\left(-2; \frac{1}{2}\right)$	$\left(\frac{1}{2}; \frac{5}{2}\right)$	$\left(\frac{5}{2}; 8\right)$	$\langle 8; \infty$
$(8-x)$	+	+	+	+	-
$(2x-5)$	-	-	-	+	+
$(x+2)$	-	+	+	+	+
$(x-0,5)$	-	-	+	+	+
	-	+	-	+	-

$$x \in \left(-2; \frac{1}{2}\right) \cup \left(\frac{5}{2}; 8\right)$$

$$30) \quad (2^x + 1)(1 - 3x)(x^2 - 1) \leq 0$$

$$(2^x + 1)(1 - 3x)(x + 1)(x - 1) \leq 0$$

$$2^x + 1 > 0$$

	$(-\infty; -1)$	$\left\langle -1; \frac{1}{3} \right\rangle$	$\left\langle \frac{1}{3}; 1 \right\rangle$	$\langle 1; \infty$
$(1-3x)$	+	+	-	-
$(x+1)$	-	+	+	+
$(x-1)$	-	-	-	+
	+	-	+	-

$$x \in \left\langle -1; \frac{1}{3} \right\rangle \cup \langle 1; \infty$$

$$31) \quad (3x^2 + 4x + 5) \left(2^x - \frac{1}{2}\right) (9x^2 - 1) \geq 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{-44}}{6} = \emptyset \Rightarrow 3x^2 + 4x + 5 > 0$$

$$(3x^2 + 4x + 5) \left(2^x - \frac{1}{2}\right) (3x - 1)(3x + 1) \geq 0$$

	$(-\infty; -1)$	$\left\langle -1; -\frac{1}{3} \right\rangle$	$\left\langle -\frac{1}{3}; \frac{1}{3} \right\rangle$	$\left\langle \frac{1}{3}; \infty \right\rangle$
$\left(2^x - \frac{1}{2}\right)$	-	+	+	+
$(3x-1)$	-	-	-	+
$(3x+1)$	-	-	+	+
	-	+	-	+

$$x \in \left\langle -1; -\frac{1}{3} \right\rangle \cup \left\langle \frac{1}{3}; \infty \right)$$

$$32) \frac{-1 + \log x}{(3 - 3^x)(x - 4)} \geq 0$$

	$(-\infty; 1)$	$(1; 4)$	$(4; 10)$	$\langle 10; \infty$
$(\log x - 1)$	-	-	-	+
$(3 - 3^x)$	+	-	-	-
$(x - 4)$	-	-	+	+
	+	-	+	-

$$x > 0$$

$$x \in (0; 1) \cup (4; 10)$$

$$33) \frac{\ln x}{(2^x - 8)(x - 5)} \leq 0$$

	$(-\infty; 1)$	$\langle 1; 3)$	$(3; 5)$	$(5; \infty)$
$(\ln x)$	-	+	+	+
$(2^x - 8)$	-	-	+	+
$(x - 5)$	-	-	-	+
	-	+	-	+

$$x > 0$$

$$x \in (0; 1) \cup (3; 5)$$

$$34) \frac{x-1}{x+2} > \frac{x+3}{x-2}$$

$$\frac{(x-1)(x-2) - (x+3)(x+2)}{(x+2)(x-2)} > 0$$

$$\frac{x^2 - 3x + 2 - x^2 - 5x - 6}{(x+2)(x-2)} > 0$$

$$\frac{-4(2x+1)}{(x+2)(x-2)} > 0$$

	$(-\infty; -2)$	$(-2; -0,5)$	$\langle -0,5; 2)$	$(2; \infty)$
$(2x+1)$	-	-	+	+
$(x+2)$	-	+	+	+
$(x-2)$	-	-	-	+
	+	-	+	-

$$x \in (-\infty; -2) \cup \left\langle -\frac{1}{2}; 2 \right)$$

$$35) \frac{2x+3}{x-6} < \frac{x+9}{x-7}$$

$$\frac{(2x+3)(x-7) - (x+9)(x-6)}{(x-6)(x-7)} < 0$$

$$\frac{2x^2 - 11x - 21 - x^2 - 3x + 54}{(x-6)(x-7)} < 0$$

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$$\frac{x^2 - 14x + 33}{(x-6)(x-7)} < 0$$

$$x_{1,2} = \frac{14 \pm 8}{2} \rightarrow x_1 = 11, x_2 = 3$$

$$\frac{(x-11)(x-3)}{(x-6)(x-7)} < 0$$

	$(-\infty; 3)$	$(3; 6)$	$(6; 7)$	$(7; 11)$	$(11; \infty)$
$(x-11)$	-	-	-	-	+
$(x-3)$	-	+	+	+	+
$(x-6)$	-	-	+	+	+
$(x-7)$	-	-	-	+	+
	+	-	+	-	+

$$x \in (3; 6) \cup (7; 11)$$

$$36) \quad \frac{3}{x+1} + \frac{7}{x+2} < \frac{6}{x-1}$$

$$\frac{3(x+2)(x-1) + 7(x^2-1) - 6(x+1)(x+2)}{(x+1)(x+2)(x-1)} < 0$$

$$\frac{3x^2 + 3x - 6 + 7x^2 - 7 - 6x^2 - 18x - 12}{(x+1)(x+2)(x-1)} < 0$$

$$\frac{4x^2 - 15x - 25}{(x+1)(x+2)(x-1)} < 0$$

$$x_{1,2} = \frac{15 \pm 25}{8} \rightarrow x_1 = 5, x_2 = -\frac{5}{4}$$

$$\frac{\left(x + \frac{5}{4}\right)(x-5)}{(x+1)(x+2)(x-1)} < 0$$

	$(-\infty; -2)$	$\left(-2; -\frac{5}{4}\right)$	$\left(-\frac{5}{4}; -1\right)$	$(-1; 1)$	$(1; 5)$	$(5; \infty)$
$(x-5)$	-	-	-	-	-	+
$\left(x + \frac{5}{4}\right)$	-	-	+	+	+	+
$(x+1)$	-	-	-	+	+	+
$(x-1)$	-	-	-	-	+	+
$(x+2)$	-	+	+	+	+	+
	-	+	-	+	-	+

$$x \in (-\infty; -2) \cup \left(-\frac{5}{4}; -1\right) \cup (1; 5)$$

$$37) \frac{5-x}{3-x} < \frac{3x-1}{2-x}$$

$$\frac{(5-x)(2-x) - (3x-1)(3-x)}{(3-x)(2-x)} < 0$$

$$\frac{10-7x+x^2+3x^2-10x+3}{(3-x)(2-x)} < 0$$

$$\frac{4x^2-17x+13}{(3-x)(2-x)} < 0$$

$$x_{1,2} = \frac{17 \pm 9}{8} \rightarrow x_1 = 1, x_2 = \frac{13}{4}$$

$$\frac{(x-1)\left(x-\frac{13}{4}\right)}{(3-x)(2-x)} < 0$$

sWidth3	$(-\infty; 1)$	$(1; 2)$	$(2; 3)$	$\left(3; \frac{13}{4}\right)$	$\left(\frac{13}{4}; \infty\right)$
$(x-1)$	-	+	+	+	+
$(x-3, 25)$	-	-	-	-	+
$(3-x)$	+	+	+	-	-
$(2-x)$	+	+	-	-	-
	+	-	+	-	+

$$x \in (1; 2) \cup \left(3; \frac{13}{4}\right)$$

$$38) \frac{7}{(x-2)(x-3)} + \frac{9}{x-3} + 1 < 0$$

$$\frac{7+9(x-2)+(x-2)(x-3)}{(x-2)(x-3)} < 0$$

$$\frac{7+9x-18+x^2-5x+6}{(x-2)(x-3)} < 0$$

$$\frac{x^2+4x-5}{(x-2)(x-3)} < 0$$

$$x_{1,2} = \frac{-4 \pm 6}{2} \rightarrow x_1 = -5, x_2 = 1$$

$$\frac{(x+5)(x-1)}{(x-2)(x-3)} < 0$$

	$(-\infty; -5)$	$(-5; 1)$	$(1; 2)$	$(2; 3)$	$(3; \infty)$
$(x-1)$	-	-	+	+	+
$(x+5)$	-	+	+	+	+
$(x-2)$	-	-	-	+	+
$(x-3)$	-	-	-	-	+
	+	-	+	-	+

$$x \in (-5; 1) \cup (2; 3)$$

$$39) \quad 2 - \frac{x-3}{x-2} \geq \frac{x-2}{x-1}$$

$$\frac{2(x-1)(x-2) - (x-3)(x-1) - (x-2)^2}{(x-2)(x-1)} \geq 0$$

$$\frac{2x^2 - 6x + 4 - x^2 + 4x - 3 - x^2 + 4x - 4}{(x-2)(x-1)} \geq 0$$

$$\frac{2x-3}{(x-2)(x-1)} \geq 0$$

	$(-\infty; 1)$	$(1; 1,5)$	$\langle 1,5; 2 \rangle$	$(2; \infty)$
$(2x-3)$	-	-	+	+
$(x-2)$	-	-	-	+
$(x-1)$	-	+	+	+
	-	+	-	+

$$x \in \left(1; \frac{3}{2}\right) \cup (2; \infty)$$

$$40) \quad \frac{2(x-4)}{(x-1)(x-7)} \geq \frac{1}{x-2}$$

$$\frac{2(x-4)(x-2) - (x-1)(x-7)}{(x-1)(x-7)(x-2)} \geq 0$$

$$\frac{2x^2 - 12x + 16 - x^2 + 8x - 7}{(x-1)(x-7)(x-2)} \geq 0$$

$$\frac{x^2 - 4x + 9}{(x-1)(x-7)(x-2)} \geq 0$$

$$x_{1,2} = \frac{4 \pm \sqrt{-20}}{2} = \emptyset \Rightarrow x^2 - 4x + 9 > 0$$

	$(-\infty; 1)$	$(1; 2)$	$(2; 7)$	$(7; \infty)$
$(x-1)$	-	+	+	+
$(x-7)$	-	-	-	+
$(x-2)$	-	-	+	+
	-	+	-	+

$$x \in (1; 2) \cup (7; \infty)$$

$$41) \quad \frac{x-1}{x+1} + \frac{x+1}{x-1} > \frac{10}{3}$$

$$\frac{3(x-1)^2 + 3(x+1)^2 - 10(x+1)(x-1)}{3(x+1)(x-1)} > 0$$

$$\frac{3x^2 - 6x + 3 + 3x^2 + 6x + 3 - 10x^2 + 10}{3(x+1)(x-1)} > 0$$

$$\frac{-4(x^2 - 4)}{3(x-1)(x+1)} > 0$$

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$$\frac{-4(x+2)(x-2)}{3(x+1)(x-1)} > 0$$

	$(-\infty; -2)$	$(-2; -1)$	$(-1; 1)$	$(1; 2)$	$(2; \infty)$
$(x+2)$	-	+	+	+	+
$(x-2)$	-	-	-	-	+
$(x-1)$	-	-	-	+	+
$(x+1)$	-	-	+	+	+
	-	+	-	+	-

$$x \in (-2; -1) \cup (1; 2)$$

$$42) \quad \frac{2x}{x^2 - 9} \leq \frac{1}{x+2}$$

$$\frac{2x(x+2) - (x-3)(x+3)}{(x-3)(x+3)(x+2)} \leq 0$$

$$\frac{2x^2 + 4x - x^2 + 9}{(x-3)(x+3)(x+2)} \leq 0$$

$$\frac{x^2 + 4x + 9}{(x-3)(x+3)(x+2)} \leq 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{-20}}{2} = \emptyset \Rightarrow x^2 + 4x + 9 > 0$$

s	$(-\infty; -3)$	$(-3; -2)$	$(-2; 3)$	$(3; \infty)$
$(x-3)$	-	-	-	+
$(x+3)$	-	+	+	+
$(x+2)$	-	-	+	+
	-	+	-	+

$$x \in (-\infty; -3) \cup (-2; 3)$$

$$43) \quad \frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$$

$$\frac{x(x-1) + x(x-2) - (x-1)(x-2)}{x(x-1)(x-2)} > 0$$

$$\frac{x^2 - x + x^2 - 2x - x^2 + 3x - 2}{x(x-1)(x-2)} > 0$$

$$\frac{(x-\sqrt{2})(x+\sqrt{2})}{x(x-1)(x-2)} > 0$$

	$(-\infty; -\sqrt{2})$	$(-\sqrt{2}; 0)$	$(0; 1)$	$(1; \sqrt{2})$	$(\sqrt{2}; 2)$	$(2; \infty)$
$(x-\sqrt{2})$	-	-	-	-	+	+
$(x+\sqrt{2})$	-	+	+	+	+	+
$n0x$	-	-	+	+	+	+
$(x-1)$	-	-	-	+	+	+
$(x-2)$	-	-	-	-	-	+
	-	+	-	+	-	+

$$x \in (-\sqrt{2}; 0) \cup (1; \sqrt{2}) \cup (2; \infty)$$

$$44) \frac{(2-x^2)(x-3)^3}{(x+1)(x^2-3x-4)} \geq 0$$

$$x_{1,2} = \frac{3 \pm 5}{2} \rightarrow x_1 = 4, x_2 = -1$$

$$\frac{(\sqrt{2}-x)(\sqrt{2}+x)(x-3)^3}{(x+1)(x+1)(x-4)} \geq 0$$

	$(-\infty; -\sqrt{2})$	$\langle -\sqrt{2}; -1 \rangle$	$(-1; \sqrt{2})$	$\langle \sqrt{2}; 3 \rangle$	$\langle 3; 4 \rangle$	$(4; \infty)$
$(\sqrt{2}-x)$	+	+	+	-	-	-
$(\sqrt{2}+x)$	-	+	+	+	+	+
$(x-3)$	-	-	-	-	+	+
$(x+1)$	-	-	+	+	+	+
$(x-4)$	-	-	-	-	-	+
	-	+	+	-	+	-

$$x \in \langle -\sqrt{2}; -1 \rangle \cup (-1; \sqrt{2}) \cup \langle 3; 4 \rangle$$

$$45) \frac{x^4 - 3x^3 + 2x^2}{x^2 - x - 30} > 0$$

$$\frac{x^2(x^2 - 3x + 2)}{x^2 - x - 30} > 0$$

$$x_{1,2} = \frac{3 \pm 1}{2} \rightarrow x_1 = 2, x_2 = 1$$

$$x_{3,4} = \frac{1 \pm 11}{2} \rightarrow x_3 = 6, x_4 = -5$$

$$\frac{x^2(x-2)(x-1)}{(x+5)(x-6)} > 0$$

	$(-\infty; -5)$	$(-5; 1)$	$(1; 2)$	$(2; 6)$	$(6; \infty)$
$(x-2)$	-	-	-	+	+
$(x-1)$	-	-	+	+	+
$(x+5)$	-	+	+	+	+
$(x-6)$	-	-	-	-	+
	+	-	+	-	+

$$x \in (-\infty; -5) \cup (1; 2) \cup (6; \infty)$$

$$46) \frac{4x^2 - 5x - 1}{2x^2 - 5x + 3} < 1$$

$$\frac{4x^2 - 5x - 1 - 2x^2 + 5x - 3}{2x^2 - 5x + 3} < 0$$

$$x_{1,2} = \frac{5 \pm 1}{4} \rightarrow x_1 = 1, x_2 = \frac{3}{2}$$

$$\frac{2(x^2 - 2)}{(x-1)\left(x - \frac{3}{2}\right)} < 0$$

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$$\frac{2(x-\sqrt{2})(x+\sqrt{2})}{(x-1)\left(x-\frac{3}{2}\right)} < 0$$

	$(-\infty; -\sqrt{2})$	$(-\sqrt{2}; 1)$	$(1; \sqrt{2})$	$\left(\sqrt{2}; \frac{3}{2}\right)$	$\left(\frac{3}{2}; \infty\right)$
$(x-\sqrt{2})$	-	-	-	+	+
$(x+\sqrt{2})$	-	+	+	+	+
$(x-1, 5)$	-	-	-	-	+
$(x-1)$	-	-	+	+	+
	+	-	+	-	+

$$x \in (-\sqrt{2}; 1) \cup \left(\sqrt{2}; \frac{3}{2}\right)$$

$$47) \quad \frac{1}{x-8} + \frac{1}{x-6} + \frac{1}{x+6} + \frac{1}{x+8} > 0$$

$$\frac{(x-6)(x+8)(x+6) + (x-8)(x+8)(x+6) + (x-8)(x+8)(x-6) + (x-8)(x-6)(x+6)}{(x+8)(x-8)(x+6)(x-6)} > 0$$

$$\frac{(x^2-36)(x+8) + (x^2-64)(x+6) + (x^2-64)(x-6) + (x^2-36)(x-8)}{(x+8)(x-8)(x-6)(x+6)} > 0$$

$$\frac{(x^2-64)(x+6+x-6) + (x^2-36)(x+8+x-8)}{(x-8)(x+8)(x-6)(x+6)} > 0$$

$$\frac{2x^3 - 128x + 2x^3 - 72x}{(x+8)(x-8)(x+6)(x-6)} > 0$$

$$\frac{4x(x^2-50)}{(x+8)(x-8)(x+6)(x-6)} > 0$$

$$\frac{4x(x-5\sqrt{2})(x+5\sqrt{2})}{(x+8)(x-8)(x+6)(x-6)} > 0$$

	$(-\infty; -8)$	$(-8; -5\sqrt{2})$	$(-5\sqrt{2}; -6)$	$(-6; 0)$	$(0; 6)$	$(6; 5\sqrt{2})$	$(5\sqrt{2}; 8)$	$(8; \infty)$
x	-	-	-	-	+	+	+	+
$(x-5\sqrt{2})$	-	-	-	-	-	-	+	+
$(x+5\sqrt{2})$	-	-	+	+	+	+	+	+
$(x-8)$	-	-	-	-	-	-	-	+
$(x+8)$	-	+	+	+	+	+	+	+
$(x-6)$	-	-	-	-	-	+	+	+
$(x+6)$	-	-	-	+	+	+	+	+
	-	+	-	+	-	+	-	+

$$x \in (-8; -5\sqrt{2}) \cup (-6; 0) \cup (6; 5\sqrt{2}) \cup (8; \infty)$$

$$48) \quad -\frac{2}{3} < \frac{3-4x}{5x+2} < \frac{3}{2}$$

$$-\frac{2}{3} < \frac{3-4x}{5x+2}$$

$$0 < \frac{9-12x+10x+4}{3(5x+2)}$$

$$0 < \frac{-2x+13}{3(5x+2)}$$

	$(-\infty; -0,4)$	$(-0,4; 6,5)$	$(6,5; \infty)$
$(13-2x)$	+	+	-
$(5x+2)$	-	+	+
	-	+	-

$$x \in (-0,4; 6,5)$$

$$\frac{3-4x}{5x+2} < \frac{3}{2}$$

$$\frac{6-8x-15x-6}{2(5x+2)} < 0$$

$$\frac{23x}{2(5x+2)} > 0$$

	$(-\infty; -0,4)$	$(-0,4; 0)$	$(0; \infty)$
$x$	-	-	+
$(5x+2)$	-	+	+
	+	-	+

$$x \in (-\infty; -0,4) \cup (0; \infty)$$

$$x \in (0; 6,5)$$

$$49) \quad 1 < \frac{3x^2 - 7x + 8}{x^2 + 1} \leq 2$$

$$1 < \frac{3x^2 - 7x + 8}{x^2 + 1}$$

$$0 < \frac{3x^2 - 7x + 8 - x^2 - 1}{x^2 + 1}$$

$$0 < \frac{2x^2 - 7x + 7}{x^2 + 1}$$

$$x^2 + 1 > 0, x_{1,2} = \frac{7 \pm \sqrt{-7}}{4} = \emptyset \Rightarrow 2x^2 - 7x + 7 > 0$$

$$x \in R$$

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$$\frac{3x^2 - 7x + 8}{x^2 + 1} \leq 2$$

$$\frac{3x^2 - 7x + 8 - 2x^2 - 2}{x^2 + 1} \leq 0$$

$$\frac{x^2 - 7x + 6}{x^2 + 1} \leq 0$$

$$x^2 + 1 > 0, x_{1,2} = \frac{7 \pm 5}{2} \rightarrow x_1 = 6, x_2 = 1$$

$$\frac{(x-6)(x-1)}{x^2 + 1} \leq 0$$

	$(-\infty; 1)$	$\langle 1; 6 \rangle$	$\langle 6; \infty$
$(x-6)$	-	-	+
$(x-1)$	-	+	+
	+	-	+

$$x \in \langle 1; 6 \rangle$$

$$x \in \langle 1; 6 \rangle$$

$$50) 1 < \frac{2x^2 - 7x - 29}{x^2 - 2x - 15} < 2$$

$$1 < \frac{2x^2 - 7x - 29}{x^2 - 2x - 15}$$

$$0 < \frac{2x^2 - 7x - 29 - x^2 + 2x + 15}{x^2 - 2x - 15}$$

$$0 < \frac{x^2 - 5x - 14}{x^2 - 2x - 15}$$

$$x_{1,2} = \frac{5 \pm 9}{2} \rightarrow x_1 = 7, x_2 = -2$$

$$x_{3,4} = \frac{2 \pm 8}{2} \rightarrow x_3 = 5, x_4 = -3$$

$$0 < \frac{(x+2)(x-7)}{(x-5)(x+3)}$$

	$(-\infty; -3)$	$(-3; -2)$	$(-2; 5)$	$(5; 7)$	$(7; \infty)$
$(x+2)$	-	-	+	+	+
$(x-7)$	-	-	-	-	+
$(x-5)$	-	-	-	+	+
$(x+3)$	-	+	+	+	+
	+	-	+	-	+

$$x \in (-\infty; -3) \cup (-2; 5) \cup (7; \infty)$$

## Nerovnice v součinném a podílovém tvaru

$$\frac{2x^2 - 7x - 29}{x^2 - 2x - 15} < 2$$

$$\frac{2x^2 - 7x - 29 - 2x^2 + 4x + 30}{x^2 - 2x - 15} < 0$$

$$\frac{-3x + 1}{x^2 - 2x - 15} < 0$$

$$x_{1,2} = \frac{2 \pm 8}{2} \rightarrow x_1 = 5, x_2 = -3$$

$$\frac{1 - 3x}{(x + 3)(x - 5)} < 0$$

	$(-\infty; -3)$	$\left(-3; \frac{1}{3}\right)$	$\left(\frac{1}{3}; 5\right)$	$(5; \infty)$
$(1-x)$	+	+	-	-
$(x-5)$	-	-	-	+
$(x+3)$	-	+	+	+
	+	-	+	-

$$x \in \left(-3; \frac{1}{3}\right) \cup (5; \infty)$$

$$x \in \left(-2; \frac{1}{3}\right) \cup (7; \infty)$$